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Resistance distance local rules

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Abstract In [D.J. Klein, Croat. Chem. Acta. **75**(2), 633 (2002)] Klein established a number of sum rules to compute the resistance distance of an arbitrary graph, especially he gave a specific set of local sum rules that determined all resistance distances of a graph (saying the set of local sum rules is complete). Inspired by this result, we give another complete set of local rules, which is simple and also efficient, especially for distance-regular graphs. Finally some applications to chemical graphs (for example the Platonic solids as well as their vertex truncations, which include the graph of Buckminsterfullerene and the graph of boron nitride hetero-fullerenoid $B_{12}N_{12}$) are made to illustrate our approach.

Keywords Resistance distance · Laplacian matrix · Distance regular graph

1 Introduction

The resistance distance is a novel distance function on a graph proposed by Klein and Randic [1]. The term resistance distance was used because of the physical interpretation: one imagines unit resistors on each edge of a graph G and takes the resistance distance between vertices i and j of G to be the effective resistance between vertices i and j, denoted by Ω_{ij}. Similar to the long recognized shortest path distance, which is denoted by $d(i, j)$, the resistance distance is also intrinsic to the graph, not only with some nice purely mathematical and physical interpretations [2,3], but with a substantial potential for chemical applications. In fact, for those two distance

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functions, the shortest-path might be imagined to be more relevant when there is corpuscular communication (along edges) between two vertices, whereas the resistance distance might be imagined to be more relevant when the communication is wave- or fluid-like. Then that chemical communication in molecules is rather wave-like suggests the utility of this concept in chemistry. So in recent years, the resistance distance was much studied in the chemical literature [4–15]. It is found that the resistance distance is closely related with many well known graph invariants, such as the connectivity index, the Balaban index, etc. This further suggests the resistance distance is worthy of study. In this paper, following the notations used in [16], G denotes a (molecular) graph, the vertex (or site) set is V and the edge (or bond) set is E, N is the number of vertices. Then *the Laplacian matrix* $L = D - A$ is $N \times N$, where A is the adjacency matrix of G and D is the diagonal matrix in which the i-th diagonal entry is Δ_i (the degree of vertex i). I denotes the identity matrix, $\Phi$is the matrix with all its elements being $1/N$. In [1], the authors gave general sum rules for resistance distances:

Theorem 1.1 ([1,16]) *For G an N-vertex connected graph and an arbitrary $N \times N$ matrix M,*

$$\sum_{i,j \in V} (LML)_{ij} \Omega_{ij} = -2tr(ML). \tag{1}$$

Here tr denote the trace operation, which sums over the diagonal elements of the matrix.

Using this general sum rule, for different particular choices of M, we can obtain different relations for resistance distances. For example, let $M = \Gamma$ and Γ^2, respectively, where Γ is a Moore-Penrose generalized inverse of L satisfied $L\Gamma = \Gamma L = I - \Phi$, then (1) led to

$$\sum_{\{i,j\} \in E} \Omega_{ij} = N - 1; \tag{2}$$

$$\sum_{i<j} \Omega_{ij} = N \sum_{k=2}^{N} \frac{1}{\mu_k}. \tag{3}$$

where $0 = \mu_1 < \mu_2 \leq \cdots \leq \mu_N$ are the eigenvalues of L.

In fact, this two particular results were long ago established by using different ways of reasoning, the first one reported in [17] and [18]; the second one, the left hand of which is the well known "Kirchhoff index," reported in [7] and [12]. In paper [16], by choosing M as the matrix O_{ab} which has all elements 0 except the (a, b)th element which is 1, Klein obtained a complete set of local rules for resistance distances, that is

Theorem 1.2 ([16]) *Let $a, b \in V$ in a connected graph. Then*

$$\Delta_a^{-1} \sum_{i,j \in n(a)} \Omega_{ij} = \sum_{i \in n(a)} \Omega_{ia} - 1, \tag{4}$$

$$\Delta_a \Delta_b \Omega_{ab} - \Delta_a \sum_{j \in n(b)} \Omega_{aj} - \Delta_b \sum_{i \in n(a)} \Omega_{ib} + \sum_{i \in n(a)} \sum_{j \in n(b)} \Omega_{ij} = 2\delta_{a \sim b}, \quad a \neq b, \tag{5}$$

where $\delta_{a\sim b}$ is 0 unless a and b are neighbors in which case it takes the values 1, where $n(a), n(b)$ denote neighbor sets of a, b respectively.

Furthermore the relations in (4) and (5) determine all the Ω_{ij}, if it is understood that $\Omega_{ij} = \Omega_{ji}$ and $\Omega_{ii} = 0$ for all $i, j \in V$. In section two, we will give another set of more local sum rules, by which we can easily get (4) and (5), so it is complete too. In Sect. 3, using these new local sum rules, we compute the resistance distances of some graphs.

2 A complete set of local sum rules for resistance distances

Before we give the main results of this section, we first introduce some notations. If α denotes a N-dimensional column vector, then α' denotes its transpose. (α, β) Denotes the ordinary vector inner product.

Theorem 2.1 *Let $G = (V, E)$ be a connected graph with $N (N \geq 2)$ vertices. Then*

(i) *For any $a, b \in V (a \neq b)$,*

$$\Delta_a \Omega_{ab} + \sum_{i \in n(a)} (\Omega_{ia} - \Omega_{ib}) = 2;$$

(ii) *For any three different vertices $a, b, c \in V$,*

$$\Delta_c (\Omega_{ca} - \Omega_{cb}) + \sum_{i \in n(c)} (\Omega_{ib} - \Omega_{ia}) = 0.$$

Proof Let $l_1, l_2, \ldots, l_N$ denote the column vectors of the Laplacian matrix L, then $l_1 + l_2 + \cdots + l_N = 0$. In view of the connectivity of the graph, the dimension of the subspace spanned by $l_1, l_2, \ldots, l_N$ is $N - 1$. So from the knowledge of linear algebra, for any $a, b \in V (a \neq b)$ there exists a vector α such that $(\alpha, l_a) = 1$, $(\alpha, l_b) = -1$ and $(\alpha, l_i) = 0$ for $i \neq a, b$. Let M be the matrix which has all elements 0 except the c-th row which is α', then

$$M = \begin{pmatrix} 0' \\ \vdots \\ \alpha' \\ \vdots \\ 0' \end{pmatrix}$$

So

$$ML = \begin{pmatrix} 0' \\ \vdots \\ \alpha' \\ \vdots \\ 0' \end{pmatrix} (l_1, l_2, \ldots, l_N) = \begin{pmatrix} 0 \cdots & \overset{\substack{a\\ \downarrow}}{0} & \cdots & \overset{\substack{b\\ \downarrow}}{0} & \cdots 0 \\ \vdots \ddots & \vdots & \ddots & \vdots & \ddots \vdots \\ 0 \cdots & 1 & \cdots & -1 & \cdots 0 \\ \vdots \ddots & \vdots & \ddots & \vdots & \ddots \vdots \\ 0 \cdots & 0 & \cdots & 0 & \cdots 0 \end{pmatrix} \leftarrow c$$

$$LML = \begin{pmatrix} l_{11} & l_{12} & \cdots & l_{1N} \\ \ldots & \ldots & \ddots & \ldots \\ l_{N1} & l_{N2} & \cdots & l_{NN} \end{pmatrix} \begin{pmatrix} 0 \cdots & 0 & \cdots & 0 & \cdots 0 \\ \vdots \ddots & \vdots & \ddots & \vdots & \ddots \vdots \\ 0 \cdots & 1 & \cdots & -1 & \cdots 0 \\ \vdots \ddots & \vdots & \ddots & \vdots & \ddots \vdots \\ 0 \cdots & 0 & \cdots & 0 & \cdots 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \cdots & \overset{\substack{a\\ \downarrow}}{l_{1c}} & \cdots & \overset{\substack{b\\ \downarrow}}{-l_{1c}} & \cdots 0 \\ 0 \cdots & l_{2c} & \cdots & -l_{2c} & \cdots 0 \\ \vdots \ddots & \vdots & \ddots & \vdots & \ddots \vdots \\ 0 \cdots & l_{Nc} & \cdots & -l_{Nc} & \cdots 0 \end{pmatrix}$$

$$\sum_{i,j \in V} (LML)_{ij} \Omega_{ij} = \sum_{i=1}^{N} l_{ic} (\Omega_{ia} - \Omega_{ib}).$$

With the realization that one has

$$-2tr(ML) = \begin{cases} 0, & c \neq a, b; \\ -2, & c = a; \\ 2, & c = b. \end{cases}$$

Recalling $l_{ii} = \Delta_i, l_{ij} = -1$ for $i \sim j$, and $l_{ij} = 0$, otherwise , so by Theorem 1.1, when $c \neq a, b$, we have

$$\Delta_c(\Omega_{ca} - \Omega_{cb}) + \sum_{i \in n(c)} (\Omega_{ib} - \Omega_{ia}) = 0,$$

when $c = a$ or $c = b$, we have

$$-\Delta_a \Omega_{ab} + \sum_{i \in n(a)} (\Omega_{ib} - \Omega_{ia}) = -2,$$

$$\Delta_b \Omega_{ab} + \sum_{i \in n(b)} (\Omega_{ib} - \Omega_{ia}) = 2.$$

So that the results are obtained. □

In fact, (ii) can be easily derived from (i), for $c \neq a$ and $c \neq b$, as we have

$$\Delta_c \Omega_{ca} + \sum_{i \in n(c)} (\Omega_{ic} - \Omega_{ia}) = 2.$$

$$\Delta_c \Omega_{cb} + \sum_{i \in n(c)} (\Omega_{ic} - \Omega_{ib}) = 2.$$

So (ii) holds naturally. Now we will show that Klein's local sum rules—Theorem 1.2 can also be deduced from (i). So by the completeness of relations in Theorem 1.2, we have the following theorem:

Theorem 2.2 *For a connected graph G, the relations*

$$\Delta_a \Omega_{ab} + \sum_{i \in n(a)} (\Omega_{ia} - \Omega_{ib}) = 2; \forall a, b \in V, \tag{6}$$

determine all Ω_{ij}, that is, this set of local sum rules is complete.

Proof For any $a \in V$, the left hand side of equality (4) is

$$\begin{aligned}\Delta_a^{-1} \sum_{i, j \in n(a)} \Omega_{ij} &= \frac{1}{2} \Delta_a^{-1} \sum_{i \in n(a)} \sum_{j \in n(a)} \Omega_{ij} \\ &= \frac{1}{2} \Delta_a^{-1} \sum_{i \in n(a)} \left(\Delta_a \Omega_{ia} + \sum_{j \in n(a)} \Omega_{aj} - 2 \right)\end{aligned}$$

$$= \frac{1}{2}\left(\sum_{i\in n(a)} \Omega_{ia} + \sum_{j\in n(a)} \Omega_{aj} - 2\right)$$
$$= \sum_{i\in n(a)} \Omega_{ia} - 1.$$

For equality (5), if a is not a neighbor of b, then

$$\sum_{i\in n(a)} \sum_{j\in n(b)} \Omega_{ij} = \sum_{i\in n(a)} \left(\sum_{j\in n(b)} \Omega_{ij}\right) = \sum_{i\in n(a)} \left(\Delta_b\Omega_{bi} + \sum_{j\in n(b)} \Omega_{bj} - 2\right)$$
$$= \Delta_b \sum_{i\in n(a)} \Omega_{bi} + \Delta_a \left(\sum_{j\in n(b)} \Omega_{bj} - 2\right)$$
$$= \Delta_b \sum_{i\in n(a)} \Omega_{bi} + \Delta_a \left(\sum_{j\in n(b)} \Omega_{aj} - \Delta_b\Omega_{ab}\right)$$
$$= \Delta_b \sum_{i\in n(a)} \Omega_{bi} + \Delta_a \sum_{j\in n(b)} \Omega_{aj} - \Delta_a\Delta_b\Omega_{ab}.$$

If a is a neighbor of b, then

$$\sum_{i\in n(a)} \sum_{j\in n(b)} \Omega_{ij} = \sum_{i\in n(a)\setminus\{b\}} \sum_{j\in n(b)} \Omega_{ij} + \sum_{j\in n(b)} \Omega_{jb}$$
$$= \sum_{i\in n(a)\setminus\{b\}} \left(\Delta_b\Omega_{bi} + \sum_{j\in n(b)} \Omega_{bj} - 2\right) + \sum_{j\in n(b)} \Omega_{jb}$$
$$= \Delta_b \sum_{i\in n(a)} \Omega_{bi} + (\Delta_a - 1)\left(\sum_{j\in n(b)} \Omega_{bj} - 2\right) + \sum_{j\in n(b)} \Omega_{jb}$$
$$= \Delta_b \sum_{i\in n(a)} \Omega_{bi} + \Delta_a \sum_{j\in n(b)} \Omega_{aj} - \Delta_a\Delta_b\Omega_{ab} + 2.$$

So the Theorem 1.2 is obtained, also the results of this theorem. □

Obviously, this set of local sum rules is more local and simpler than Klein's, so it can simplify the computing of resistance distances, especially for some high symmetry graphs, in the following section we will give some applications.

3 Specialized results for graphs with some degree of symmetry

In this part, when we say distance and diameter, they are in shortest-path meaning.

Definition 3.1 ([19]) *A distance regular graph is a regular connected graph with degree Δ and diameter D, such that there are natural numbers*

$$b_0 = \Delta, b_1, \ldots, b_{D-1}; c_1 = 1, c_2, \ldots, c_D,$$

for each pair $i, j \in V$ satisfying $d(i, j) = k$, we have

(i) $c_k = |n(j) \cap n_{k-1}(i)|$, *where* $n_m(i) = \{a \in V|d(a, i) = m\}, 1 \leq k \leq D$;
(ii) $b_k = |n(j) \cap n_{k+1}(i)|, 0 \leq k \leq D - 1$.

The array $\{b_0 = \Delta, b_1, \ldots, b_{D-1}; c_1 = 1, c_2, \ldots, c_D\}$ is the intersection array.

For the high symmetry of the distance regular graphs, the resistance distances between vertices depend only on the distance k between these vertices, so in the following, we use Ω_k's instead of Ω_{ij}'s.

Theorem 3.1 *Let G be a distance regular graph with intersection array $\{b_0{=}\Delta, b_1, \ldots, b_{D-1}; c_1 = 1, c_2, \ldots, c_D\}$. Then we have recursion relations as follows:*

$$\Omega_{k+1} = \frac{(c_k + b_k)\Omega_k - c_k\Omega_{k-1} - 2/N}{b_k}, \quad k = 1, \ldots, D - 1.$$

where $\Omega_0 = 0, \Omega_1 = 2(N - 1)/N\Delta$.

Proof Obviously $\Omega_0 = 0$, and by equality (2) in Sect. 1, we have $N\Delta/2\Omega_1 = N - 1$, so $\Omega_1 = 2(N - 1)/N\Delta$. Now for any $i, j \in V$ with $d(i, j) = k$, $|n(i) \cap n_{k-1}(j)| = c_k$, $|n(i) \cap n_{k+1}(j)| = b_k$, $|n(i) \cap n_k(j)| = \Delta - b_k - c_k$. So by the local sum rules of Theorem 2.2, we easily get

$$\Delta\Omega_k + \Delta\Omega_1 - c_k\Omega_{k-1} - b_k\Omega_{K+1} - (\Delta - c_k - b_k)\Omega_k = 2;$$

$$\Omega_{k+1} = \frac{(c_k + b_k)\Omega_k - c_k\Omega_{k-1} - 2/N}{b_k}.$$

□

Since the regular polyhedra are distance regular graphs, using the above theorem, we easily compute the Ω_k's, see the following Table 1. The results are the same as in Klein's paper [16], but we get them from the general recursion relations for distance regular graphs, it is much easier.

Now we consider using the Theorem 2.2 for less symmetric graphs, the first example is the 14-vertex rhombic dodecahedron of Fig. 1. We also classify the resistance distances by their distance k. It is easy to see that:

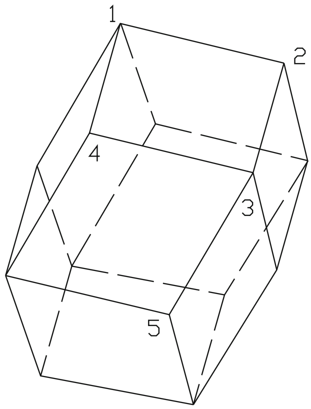
(i) For $k = 1$ and $k = 3$, there is only one class, denoted by Ω_1 and Ω_3, respectively;
(ii) If $k = 2$, there are three different classes, denoted by Ω_{13}, Ω_{24}, and Ω_{25} respectively.

Then the local sum rules of Theorem 2.2 yield

Table 1 Resistance distances for the regular polyhedra

Polyhedron	N	Δ	d	Intersection array	Ω_k
Tetrahedron	4	3	1	{3;1}	$\Omega_1 = 1/2$
Octahedron	6	4	2	{4,1;1,4}	$\Omega_1 = 5/12$, $\Omega_2 = 1/2$
Cube	8	3	3	{3,2,1;1,2,3}	$\Omega_1 = 7/12$, $\Omega_2 = 3/4$, $\Omega_3 = 5/6$
Icosahedron	12	5	3	{5,2,1;1,2,5}	$\Omega_1 = 11/30$, $\Omega_2 = 7/15$, $\Omega_3 = 1/2$
Dodecahedron	20	3	5	{3,2,1,1,1;1,1,1,2,3}	$\Omega_1 = 19/30$, $\Omega_2 = 9/10$, $\Omega_3 = 16/15$ $\Omega_4 = 17/15$, $\Omega_5 = 7/6$

Fig. 1 The rhombic dodecachedron



$$\begin{cases} \Omega_1 = 13/24 \\ 3\Omega_1 + 3\Omega_1 - 2\Omega_{13} = 2 \\ 4\Omega_1 + 4\Omega_1 - 2\Omega_{24} - \Omega_{25} = 2 \\ 3\Omega_{24} + 3\Omega_1 - 2\Omega_1 - \Omega_3 = 2 \\ 4\Omega_{13} + 4\Omega_1 - 2\Omega_1 - 2\Omega_3 = 2 \end{cases}$$

So we have

$$\Omega_1 = 13/24,\ \Omega_{13} = 5/8,\ \Omega_{24} = 3/4,\ \Omega_{25} = 5/6,\ \Omega_3 = 19/24.$$

This is also an example in [16], but there Ω_{24}, Ω_{25} are not given, and $\Omega_3 = 21/32$, which is incorrect.

Now we consider the truncated Platonic solids (see Fig. 2, the vertex labels are utilized in the following). The resistance distances for the truncated Platonic solids have been considered in [20], but the results in there are not complete. It is clear that the truncated Platonic solids are vertex transitive. So we only need to calculate $\Omega'_{a,b}s$, where a is a fixed point (in Fig. 2, is denoted by '0′'), $b \in V$. In the following, $\Omega_{a,b}$ simplified to Ω_b. As above, here we also classify the resistance distances by their distance k.

(1) *The truncated tetrahedron*(see Fig. 2a).

It is easy to see, for $k = 1$ and $k = 3$, there are two different classes, denoted by Ω_{11}, Ω_{12}, Ω_{31}, and Ω_{32}, respectively; for $k = 2$, only one class, denoted by Ω_2. So

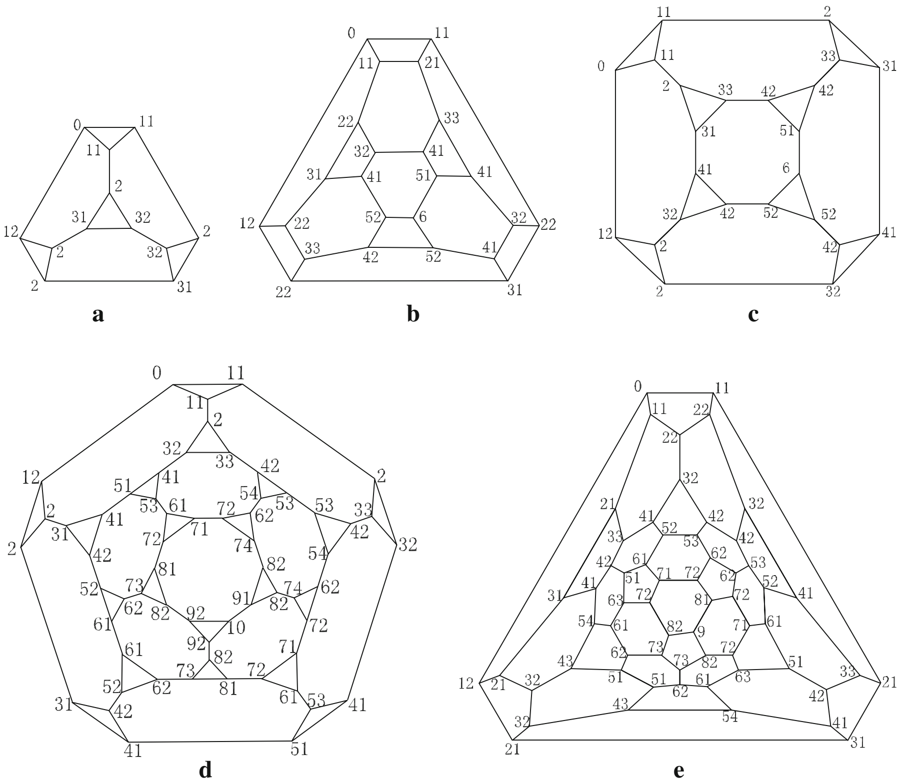


Fig. 2 The truncated platonic solids

by (2) and Theorem 2.2, we have

$$\begin{cases} 2\Omega_{11} + \Omega_{12} = 2 - 1/6 \\ 2\Omega_{11} - \Omega_2 = 1/6 \\ 3\Omega_{12} - 2\Omega_2 = 1/6 \\ 2\Omega_2 - \Omega_{12} - \Omega_{31} = 1/6 \\ 3\Omega_2 - \Omega_{11} - \Omega_{31} - \Omega_{32} = 1/6. \end{cases}$$

Then we derive

$$\Omega_{11} = 17/30,\ \Omega_{12} = 21/30;\ \Omega_2 = 29/30;\ \Omega_{31} = 32/30,\ \Omega_{32} = 33/30.$$

(2) *The truncated octahedron* (*the graph of boron nitride hetero-fullerenoid* $B_{12}N_{12}$) (see Fig. 2b)

For this graph, we find

(i) For k=3, there are three different classes, denoted by Ω_{31}, Ω_{32}, and Ω_{33}, respectively;

(ii) For $k = 6$, there is only one class, denoted by Ω_6;

(iii) For other cases, that is k=1, 2, 4, 5, each has two different classes, denoted by Ω_{k1}, Ω_{k2}, respectively. Also from Theorem 2.2 and (2), we can get the following equations:

$$\begin{cases} 2\Omega_{11} + \Omega_{12} = 2 - 1/12 \\ 3\Omega_{11} - \Omega_{21} - \Omega_{22} = 1/12 \\ 3\Omega_{12} - 2\Omega_{22} = 1/12 \\ 3\Omega_{21} - 2\Omega_{11} - \Omega_{33} = 1/12 \\ 3\Omega_{22} - \Omega_{12} - \Omega_{31} - \Omega_{33} = 1/12 \\ 3\Omega_{31} - 2\Omega_{22} - \Omega_{41} = 1/12 \\ 3\Omega_{32} - \Omega_{22} - 2\Omega_{41} = 1/12 \\ 3\Omega_{33} - 2\Omega_{22} - \Omega_{42} = 1/12 \\ 3\Omega_{41} - \Omega_{32} - \Omega_{33} - \Omega_{51} = 1/12 \\ 3\Omega_{42} - \Omega_{33} - 2\Omega_{52} = 1/12 \\ 3\Omega_{51} - 2\Omega_{41} - \Omega_6 = 1/12 \\ 3\Omega_{52} - \Omega_{41} - \Omega_{42} - \Omega_6 = 1/12. \end{cases}$$

So we get

$$\Omega_{11} = 625/1008, \Omega_{12} = 682/1008, \Omega_{21} = 810/1008, \Omega_{22} = 981/1008,$$
$$\Omega_{31} = 1081/1008, \Omega_{33} = 1096/, 1008\Omega_{32} = 1153/1008, \Omega_{41} = 1197/1008,$$
$$\Omega_{42} = 1242/1008, \Omega_{51} = 1258/1008, \Omega_{52} = 1273/1008, \Omega_6 = 1296/1008.$$

In [20], Ω_6 is missing.

(3) *The truncated cube*(see Fig. 2c)

For this graph, we can find

(i) For $k = 3$, there are three different classes, denoted by Ω_{31}, Ω_{32}, and Ω_{33}, respectively;

(ii) For $k = 2, k = 6$, there is only one class, denoted by Ω_2, Ω_6;

(iii) For $k = 1, 4, 5$, each has two different classes, denoted by Ω_{k1}, Ω_{k2}, respectively. So we get the following equations:

$$\begin{cases} 2\Omega_{11} + \Omega_{12} = 2 - 1/12 \\ 2\Omega_{11} - \Omega_2 = 1/12 \\ 3\Omega_{12} - 2\Omega_2 = 1/12 \\ 3\Omega_2 - \Omega_{11} - \Omega_{31} - \Omega_{33} = 1/12 \\ 2\Omega_2 - \Omega_{12} - \Omega_{32} = 1/12 \\ 3\Omega_{31} - \Omega_2 - \Omega_{33} - \Omega_{41} = 1/12 \\ 3\Omega_{32} - \Omega_2 - \Omega_{41} - \Omega_{42} = 1/12 \\ 3\Omega_{41} - \Omega_{32} - \Omega_{31} - \Omega_{42} = 1/12 \\ 3\Omega_{42} - \Omega_{32} - \Omega_{41} - \Omega_{52} = 1/12 \\ 2\Omega_{42} - \Omega_{33} - \Omega_{51} = 1/12 \\ 3\Omega_{51} - 2\Omega_{42} - \Omega_6 = 1/12. \end{cases}$$

Solving those equations, we get

$$\Omega_{11} = 35/60, \Omega_{12} = 45/60, \Omega_2 = 65/60, \Omega_{31} = 77/60,$$
$$\Omega_{33} = 78/60, \Omega_{32} = 80/60, \Omega_{41} = 83/60,$$
$$\Omega_{42} = 87/60, \Omega_{51} = 91/60, \Omega_{52} = 93/60, \Omega_6 = 94/60.$$

For this graph, in [20], Ω_6 is also missing.

(4) *The truncated dodecahedron* (see Fig. 2d)

The diameter of this graph is 10. After careful observation, we find

(i) For $k = 3$, there are three different classes, denoted by Ω_{31}, Ω_{32}, and Ω_{33}, respectively;
(ii) For $k = 2, 10$, there is only one class, denoted by Ω_2, Ω_{10};
(iii) For $k = 5, 7$, each has four different classes, denoted by Ω_{k1-4};
(iv) For $k = 1, 4, 6, 8, 9$, each has two different classes, denoted by Ω_{k1}, Ω_{k2}, respectively. So we have

$$\begin{cases} 2\Omega_{11} + \Omega_{12} = 2 - 1/30, 2\Omega_{11} - \Omega_2 = 1/30 \\ 3\Omega_{12} - 2\Omega_2 = 1/30 \\ 2\Omega_2 - \Omega_{12} - \Omega_{31} = 1/30 \\ 3\Omega_2 - \Omega_{11} - \Omega_{32} - \Omega_{33} = 1/30 \\ 3\Omega_{31} - \Omega_2 - \Omega_{41} - \Omega_{42} = 1/30 \\ 3\Omega_{41} - \Omega_{32} - \Omega_{51} - \Omega_{53} = 1/30 \\ 3\Omega_{41} - \Omega_{31} - \Omega_{42} - \Omega_{51} = 1/30 \\ 3\Omega_{42} - \Omega_{33} - \Omega_{53} - \Omega_{54} = 1/30 \\ 3\Omega_{42} - \Omega_{31} - \Omega_{41} - \Omega_{52} = 1/30 \\ 3\Omega_{51} - 2\Omega_{41} - \Omega_{53} = 1/30 \\ 2\Omega_{53} - \Omega_{42} - \Omega_{54} = 1/30 \\ 3\Omega_{53} - \Omega_{41} - \Omega_{51} - \Omega_{61} = 1/30 \\ 3\Omega_{54} - \Omega_{42} - \Omega_{53} - \Omega_{62} = 1/30 \\ 3\Omega_{61} - \Omega_{53} - \Omega_{71} - \Omega_{72} = 1/30 \\ 3\Omega_{62} - \Omega_{52} - \Omega_{61} - \Omega_{73} = 1/30 \\ 3\Omega_{62} - \Omega_{54} - \Omega_{72} - \Omega_{74} = 1/30 \\ 3\Omega_{71} - \Omega_{61} - 2\Omega_{72} = 1/30 \\ 3\Omega_{72} - \Omega_{61} - \Omega_{71} - \Omega_{81} = 1/30 \\ 3\Omega_{73} - \Omega_{62} - \Omega_{81} - \Omega_{82} = 1/30 \\ 3\Omega_{82} - \Omega_{74} - \Omega_{82} - \Omega_{91} = 1/30 \\ 3\Omega_{82} - \Omega_{73} - \Omega_{81} - \Omega_{92} = 1/30 \\ 3\Omega_{91} - 2\Omega_{82} - \Omega_{10} = 1/30. \end{cases}$$

Then we obtain

$$\Omega_{11} = 267/450, \Omega_{12} = 351/450, \Omega_2 = 519/450, \Omega_{32} = 635/450$$
$$\Omega_{33} = 640/450, \Omega_{31} = 672/450, \Omega_{41} = 731/450, \Omega_{42} = 751/450$$
$$\Omega_{51} = 755/450, \Omega_{53} = 788/450, \Omega_{54} = 810/450, \Omega_{52} = 835/450$$

$$\Omega_{61} = 863/450, \Omega_{62} = 876/450, \Omega_{71} = 890/450, \Omega_{72} = 896/450$$
$$\Omega_{74} = 907/450, \Omega_{73} = 915/450, \Omega_{81} = 920/450, \Omega_{82} = 934/450$$
$$\Omega_{91} = 946/450, \Omega_{92} = 952/450, \Omega_{10} = 955/450.$$

(5) *The truncated icosahedron* (*the graph of Buckminsterfullerene*) (see Fig. 2e) The diameter of this graph is 9. After careful observation, we find

- (i) For $k = 3, 4, 6, 7$, there are three different classes, denoted by $\Omega_{k1}, \Omega_{k2}, \Omega_{k3}$;
- (ii) For $k = 9$, there is only one class, denoted by Ω_9;
- (iii) For $k = 5$, there are four different classes, denoted by $\Omega_{k,1-4}$;
- (iv) For $k = 1, 2, 8$, each has two different classes, denoted by Ω_{k1}, Ω_{k2}, respectively.

$$\begin{cases} 2\Omega_{11} + \Omega_{12} = 2 - 1/30, 3\Omega_{12} - 2\Omega_{21} = 1/30 \\ 3\Omega_{11} - \Omega_{21} - \Omega_{22} = 1/30 \\ 3\Omega_{21} - \Omega_{12} - \Omega_{31} - \Omega_{32} = 1/30 \\ 3\Omega_{21} - \Omega_{11} - \Omega_{31} - \Omega_{33} = 1/30 \\ 2\Omega_{22} - \Omega_{11} - \Omega_{32} = 1/30 \\ 3\Omega_{31} - 2\Omega_{21} - \Omega_{41} = 1/30 \\ 2\Omega_{32} - \Omega_{21} - \Omega_{43} = 1/30 \\ 3\Omega_{33} - \Omega_{21} - \Omega_{42} - \Omega_{41} = 1/30 \\ 3\Omega_{41} - \Omega_{32} - \Omega_{33} - \Omega_{52} = 1/30 \\ 3\Omega_{41} - \Omega_{31} - \Omega_{42} - \Omega_{54} = 1/30 \\ 3\Omega_{42} - \Omega_{33} - \Omega_{41} - \Omega_{51} = 1/30 \\ 3\Omega_{42} - \Omega_{32} - \Omega_{42} - \Omega_{53} = 1/30 \\ 2\Omega_{51} - \Omega_{43} - \Omega_{62} = 1/30 \\ 3\Omega_{51} - \Omega_{42} - \Omega_{61} - \Omega_{63} = 1/30 \\ 3\Omega_{52} - \Omega_{41} - \Omega_{53} - \Omega_{61} = 1/30 \\ 3\Omega_{61} - \Omega_{54} - \Omega_{62} - \Omega_{63} = 1/30 \\ 3\Omega_{61} - \Omega_{52} - \Omega_{51} - \Omega_{71} = 1/30 \\ 3\Omega_{62} - \Omega_{51} - \Omega_{61} - \Omega_{73} = 1/30 \\ 3\Omega_{63} - \Omega_{51} - \Omega_{61} - \Omega_{72} = 1/30 \\ 3\Omega_{72} - \Omega_{62} - \Omega_{71} - \Omega_{81} = 1/30 \\ 3\Omega_{72} - \Omega_{63} - \Omega_{71} - \Omega_{82} = 1/30 \\ 3\Omega_9 - 2\Omega_{82} - \Omega_{81} = 1/30. \end{cases}$$

Then we get

$$\Omega_{11} = 16273/25080, \Omega_{12} = 16778/25080, \Omega_{21} = 24749/25080$$
$$\Omega_{22} = 23234/25080, \Omega_{31} = 27274/25080, \Omega_{32} = 29359/25080$$
$$\Omega_{33} = 29864/25080, \Omega_{41} = 31488/25080, \Omega_{42} = 32519/25080$$
$$\Omega_{43} = 33133/25080, \Omega_{54} = 33835/25080, \Omega_{52} = 34405/25080$$

$$\begin{aligned}&\Omega_{53} = 34843/25080, \Omega_{51} = 35369/25080, \Omega_{61} = 36048/25080\\&\Omega_{63} = 36704/25080, \Omega_{62} = 36769/25080, \Omega_{71} = 37534/25080\\&\Omega_{72} = 37859/25080, \Omega_{73} = 38054/25080, \Omega_{81} = 38438/25080\\&\Omega_{82} = 38503/25080, \Omega_{9} = 38760/25080.\end{aligned}$$

For this graph, in [20], Ω_9 is missing.

We know the truncated polyhedra are all Archimedean polyhedra. Similarly for other Archimedean polyhedra, such as cuboctahedron, icosidodecahedron, snub cube, ect., we can also calculate the resistance distances by our method.

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